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ON THE USE OF NEGATIVE TIME-LIKE STATES

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ABSTRACT

In this paper it is shown that the analytic continuation of the quasi-elastic scattering amplitude into the different channels can lead to negative energy time-like states. Its advantages and difficulties are discussed.

РЕЗЮМЕ

В статье показывается, что аналитическое продолжение амплитуды квази-эластичного рассеяния в различные каналы может привести к времени-подобному состоянию с отрицательной энергией. Рассматриваются преимущества и недостатки этого явления.

KIVONAT

A cikkben megmutatjuk, hogy a kvázi-elasztikus szórási amplitudó analitikus folytatása különböző csatornákba negativ energiáju, időszerű állapotokhoz vezethet. Ennek előnyeit és hátrányait diszkutáljuk.

1. INTRODUCTION

The crossing principle is one of the basic assumption of analytic theories, and the behaviour of a two-particle scattering amplitude under crossing was examined in detail[1,2]. In these examinations there appeared negative time-like states, but their extensive use is connected with the introduction of "crossed channel expansions" of the scattering amplitude[3]. In an earlier paper[4] it was shown that the analytic continuation of a quasielastic scattering amplitude can lead to negative time-like one-particle states. That examination was restricted only to one crossed channel. Here we extend it for all possibilities, correcting as well a smaller mistake.

After summarizing in sect.2. the kinematics we use, we examine the crossing properties in sect.3. The results and some of their consequences are discussed in sect.4.

2. ONE- AND TWO-PARTICLE STATES

A one-particle state $|p_\mu, s, \lambda\rangle$ is defined as the eigenvector of the operators

$$P_\mu |p_\mu, s, \lambda\rangle = p_\mu |p_\mu, s, \lambda\rangle \; ; \quad W_\mu^2 |p_\mu, s, \lambda\rangle = p_\mu^2 \, s(s+1) |p_\mu, s, \lambda\rangle$$

$$\left.\begin{aligned} W_o |p_\mu, s, \lambda\rangle &= |\underline{p}| \lambda |p_\mu, s, \lambda\rangle \quad &\text{if} \quad p_i \neq 0 \\ W_3 |m, \underline{O}, s, \lambda\rangle &= m\lambda |m, \underline{O}, s, \lambda\rangle \quad &\text{if} \quad p_i = 0 \end{aligned}\right\} \qquad /2.1/$$

with eigenvalues specificed above. A "moving state" is created by a pure boost from a "rest state":

$$|p, s, \lambda\rangle = e^{-i\varphi M_3} e^{-i\vartheta M_2} e^{-i\alpha N_3} |m, \underline{O}, s, \lambda\rangle \qquad /2.2/$$

where $\alpha \geq 0$, $0 \leq \vartheta \leq \pi$, $0 \leq \varphi \leq 2\pi$; M_i and N_i are the generators of the Lorentz group. Eq./2.2./ can be considered as the definition of our phase convention. A two-particle state $|p_1,s_1,\lambda_1; p_2,s_2,\lambda_2>$ is defined as an element of a direct product space:

$$|p_1,s_1,{}_1; p_2,s_2,\lambda_2> = |p_1,s_1,\lambda_1> \otimes (-1)^{s_2} |p_2,s_2,\lambda_2> \qquad /2.3/$$

the $(-1)^{s_2}$ phase factor before the second particle is a matter of convention. If the center of mass of the two particles is at rest, we may write

$$|p_1,s_1,\lambda_1; p_2,s_2,\lambda_2>_{CM} = e^{-i\varphi M_3'} e^{-i\vartheta M_2'} e^{-i\alpha_1 N_3'} |m_1,\underline{0},s_1,\lambda_1>$$

$$\cdot (-1)^{s_2} e^{-i(\varphi+\pi)M_3''} e^{-i(\pi-\vartheta)M_2''} e^{-i\alpha_2 N_3''} |m_2,\underline{0},s_2,\lambda_2> \qquad /2.4/$$

The primed operators act onto particle 1, the double-primed onto particle 2; the primed and double primed operators commute. We may rearrange the order of the operators as follows:

$$|......>_{CM} = e^{-i\varphi(M_3'+M_3'')} e^{-i\vartheta(M_2'+M_2'')} e^{-i\alpha_1 N_3'+i\alpha_2 N_3''} e^{-i\pi M_2''} e^{-i\pi M_3''}$$

$$\cdot |m_1,\underline{0},s_1,\lambda_1> \otimes (-1)^{s_2} |m_2,\underline{0},s_2\lambda_2>$$

$$= e^{-i\varphi M_3^+} e^{-i\vartheta M_2^+} e^{-i\alpha_1 N_3'+i\alpha_2 N_3''} e^{-i\pi\lambda_2} (-1)^{s_2} (-1)^{-s_2+\lambda_2}$$

$$|m_1,\underline{0},s_1,\lambda_1> \otimes |m_2,\underline{0},s_2,-\lambda_2> \qquad /2.5/$$

Denoting the total four-momentum of the two-particle state as $(\sqrt{s}, \underline{0})$ provided $s=(p_1+p_2)^2>0$, the usual kinematics gives

$$\text{ch}\,\alpha_1 = \frac{s+m_1^2-m_2^2}{2\sqrt{s}\,m_1}\ , \quad \text{sh}\,\alpha_1 = \frac{\sqrt{\left[s-(m_1-m_2)^2\right]\left[s-(m_1+m_2)^2\right]}}{2\sqrt{s}m_1} \qquad /2.6/$$

$$\alpha_2 = \alpha_1\ , \quad 1 \leftrightarrow 2$$

Here $\alpha_1, \alpha_2 > 0$, and for the sake of definiteness we suppose $m_1 > m_2$ in the sequel. If we introduce the notations

$$\beta' = (\alpha_1 + \alpha_2)|2 \quad , \quad \alpha = (\alpha_1 - \alpha_2)|2$$

$$N_3' \pm N_3'' = N_3^{\pm} \qquad /2.7/$$

we may write

$$|\ldots\rangle_{CM} = e^{-i\varphi M_3^+} e^{-i\vartheta M_2^+} e^{-i\alpha N_3^+} e^{-i\beta N_3^-} |m_1, \underline{O}, s_1, \lambda_1;\ m_2, \underline{O}, s_2, -\lambda_2\rangle \qquad /2.8/$$

The + type transformations generate the movement of the center of mass, accordingly they do not alter the value of s; the − type one gives velocity of equal magnitude but of opposite direction to the two particles. Eqs. /2.6/ and /2.7/ yield

$$e^{\alpha} = \frac{M\sqrt{s-\mu^2} - \mu\sqrt{s-M^2}}{2\sqrt{s m_1 m_2}} \quad , \quad e^{\beta} = \frac{\sqrt{s-\mu^2} - \sqrt{s-M^2}}{2\sqrt{m_1 m_2}} \qquad /2.9/$$

i.e. $\alpha < 0$ $\beta > 0$; and $\mu = m_1 - m_2$, $M = m_1 + m_2$.

A scattering amplitude of positive energy particles in the CMS now can have the form: $f(s,t) =$

$$= \langle p_3, s_3, \lambda_3;\ p_4, s_4, \lambda_4 | \tau | p_1, s_1, \lambda_1;\ p_2, s_2, \lambda_2 \rangle =$$

$$= \langle m_3, \underline{O}, s_3, \lambda_3;\ m_4, \underline{O}, s_4, -\lambda_4 | e^{i\beta' N_3^-} e^{i\alpha' N_3^+} e^{-i\vartheta M_2^+} \tau\ e\ .$$

$$.\ e^{-i\alpha N_3^+} e^{-i\beta N_3^-} |m_1, \underline{O}, s_1, \lambda_1;\ m_2, \underline{O}, s_2, -\lambda_2\rangle \qquad /2.10/$$

where $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, $u = (p_1 - p_4)^2$,

$$\cos\vartheta = \frac{s(t-u) - (m_1^2-m_2^2)(m_3^2-m_4^2)}{\left\{\left(s-(m_1-m_2)^2\right)\left(s-(m_1+m_2)^2\right)\left(s-(m_3-m_4)^2\right)\left(s-(m_3-m_4)^2\right)\right\}^{1/2}}$$

ϑ is the scattering angle in the s-channels CMS. In the sequel we restrict ourselves to $m_1=m_3$, $m_2=m_4$ processes.

3. THE CROSSING

As usual, we suppose the scattering amplitude to be real analytic function of two complex variables; s,t, $u=s-t+2(m_1^2+m_2^2)$. The physical domains of this amplitude on the Mandelstam plane are shown in Fig.1. The boundary of the domains are the $|\cos\vartheta|=1$ lines. The physical amplitude in the appropriate regions is defined as

$$\lim_{\varepsilon,\delta\to 0} f(s+i\varepsilon,\ t-i\delta) = f^s(s,t)$$

$$\lim_{\varepsilon,\delta\to 0} f(s-i\varepsilon,\ t+i\delta) = f^t(s,t) \qquad /3.1/$$

$$\lim_{\varepsilon,\delta\to 0} f(s-i\varepsilon,\ t-i\delta) = f^4(s,t) \qquad \text{where} \quad \varepsilon>0, \quad \delta>0$$

We would like to continue analytically the amplitude from the s channel to the t and u channels. Hence, coming from $s+i\varepsilon$, $t-i\delta$, we have to cross somewhere the real s-t plane. This point is chosen[1] inside the region $t > 0$ and $\cos\vartheta > -1$ /e.g. the point c in Fig.1. is good/. We choose the way $A \to E'$ or $A \to E''$ depending on whether we are going to the u or to the t channel. The path in the s and t plane are shown in Figs.2.

How do the variables α,β, behave during the continuation? To see it, it is better to examine the behaviour of some frequently appearing factors. The result, which can be read off from Figs.2. are summarized in Table 1. For convenience we draw the path on the $M^2\mu_0^2$-su plane as well in Figs.3.

Table 1.

	s -channel	u-channel	t-channel
$\sqrt{s-\mu^2}$	$\sqrt{\lvert s-\mu^2\rvert}$	$-i\sqrt{\lvert s-\mu^2\rvert}$	$-i\sqrt{\lvert s-\mu^2\rvert}$
$\sqrt{s-M^2}$	$\sqrt{\lvert s-M^2\rvert}$	$i\sqrt{\lvert s-M^2\rvert}$	$i\sqrt{\lvert s-M^2\rvert}$
$\sqrt{s}$	$\sqrt{\lvert s\rvert}$	$\begin{cases} \sqrt{\lvert s\rvert} & \text{if } s>0 \\ -i\sqrt{\lvert s\rvert} & \text{if } s<0 \end{cases}$	$-i\sqrt{\lvert s\rvert}$
$\sqrt{-t}$	$\sqrt{\lvert t\rvert}$	$\sqrt{\lvert t\rvert}$	$i\sqrt{\lvert t\rvert}$
$\sqrt{M^2\mu^2 - su}$	$\sqrt{\lvert M^2\mu^2 - su\rvert}$	$\sqrt{\lvert M^2\mu^2 - su\rvert}$	$i\sqrt{\lvert M^2\mu^2 - su\rvert}$

Accordingly, the domains of the variables in question are as follows:

for α (c.f.eq.2.9a) $\quad \ln\sqrt{\frac{M-\mu}{M+\mu}} < \alpha \leq 0 \qquad$ in the s channel

$$\alpha = \tilde{\alpha} - i\pi/2, \qquad 0 \leq \tilde{\alpha} < \infty \qquad \mu^2 \geq s > 0 \tag{3.2}$$

$$\tilde{\alpha} = \ln\left(\mu\sqrt{M^2-s} + M\sqrt{\mu^2-s}\right) - \ln\sqrt{s}\left(M^2-\mu^2\right)$$

for β (c.f.eq.2.9b) $\quad 0 \leq \beta < \infty$ $\quad$ in the s channel

$$\beta = \tilde{\beta} + i\pi/2 \qquad 0 \geq \tilde{\beta} > -\infty \qquad \mu^2 \geq s \tag{3.3}$$

$$\tilde{\beta} = \ln\left(\sqrt{M^2-s} - \sqrt{\mu^2-s}\right) - \ln\sqrt{M^2-\mu^2}$$

for (c.f.eq.2.10)

$$0 \leq \vartheta \leq \pi \quad \begin{cases} \text{in the s channel} \\ \text{" " u " if } s>0 \end{cases}$$

$$\mathrm{Re}\,\vartheta=0 \quad 0>\mathrm{Jm}\,\vartheta > -\infty \quad \text{" " u " if } s<0$$

$$\mathrm{Re}\,\vartheta=\pi \quad 0>\mathrm{Jm}\,\vartheta > -\infty \quad \text{" " t "}$$

Because of the singularity in α at s=o, we restricted /3.2/ for $s > o$, c.f. below.

That form of the scattering amplitude, what is given by eq./2.10/, is valid only for $s > M^2$. In the region $\mu^2>s>0$ it can be written as $f(s,t) =$

$$\langle m_1, \underline{0}, s_3, \lambda_3; m_2, \underline{0}, s_4, -\lambda_4 \; e^{i\left(i\frac{\pi}{2}\right)\left(N_3^- - N_3^+\right)} e^{i\tilde{\beta}N_3^-} e^{i\tilde{\alpha}N_3^+} \,.$$

$$\cdot\, \tau\, e^{-i\vartheta M_2^+} e^{-i\tilde{\alpha}N_3^+} e^{-i\tilde{\beta}N_3^-} e^{-i\left(\frac{i\pi}{2}\right)\left(N_3^- - N_3^+\right)} |m_1, \underline{0}, s_1, \lambda_1; m_2, \underline{0}, s_2, -\lambda_2\rangle \tag{3.5}$$

We may write further

$$\exp\left(-i\left(i\frac{\pi}{2}\right)\left(N_3^- - N_3^+\right)\right) = \exp\left(-i\left(-i\pi N_3''\right)\right) \tag{3.6}$$

the operator exp $\left(-i\left(-i\pi N_3''\right)\right)$ acts only onto the second particle; it changes the sign of the energy and of the helicity. Out of these the first is almost trivial, the second can be shown as follows: the $-\lambda_2$ value in the rest frame is measured by W_3 /c.f. eq.2.1/. As $[N_3, W_3]$ $|p_0, \underline{0}, s, \lambda\rangle = -iW_0 \; |p_0, \underline{0}, s, \lambda\rangle = 0$, i.e. the eigenvalue of W_3, what is $p_0\lambda$, must be the same. Since the sign of p_0 changes, so does the sign of λ .

Hence

$$e^{-i(-i\pi N_3'')}|m_1,\underline{O},s_1,\lambda_1;\ m_2,\underline{O},s_2,-\lambda_2\rangle = \eta_2(-1)^{s_2-\lambda_2}|m_1,\underline{O},s_1,\lambda_1;-m_2,\underline{O},s_2,\lambda_2\rangle \qquad /3.7/$$

where η_2 is some phase factor. Let us allow to act $\exp(-i\tilde\beta N_3^-)$ as well:

$$\exp(-i\tilde\beta N_3^-)|m_1,\underline{O},s_1,\lambda_1;\ -m_2,\underline{O},s_2,\lambda_2\rangle =$$

$$= (-1)^{s_2-\lambda_2}|m_1(ch\tilde\beta,0,0,-sh\tilde\beta),\ s_1,\lambda_1;\ -m_2(ch\tilde\beta,0,0,sh\tilde\beta),\ s_2,-\lambda_2\rangle \qquad /3.8/$$

$$= (-1)^{s_2-\lambda_2}|\tilde p_1,s_1,\lambda_1;\ \tilde p_2,s_2,-\lambda_2\rangle$$

where we used the identity

$$\exp(-i\tilde\beta N_3^-) = \exp(-i\tilde\beta N_3')\exp(i\pi M_2'')\exp(-i\tilde\beta N_3'')\exp(-i\pi M_2'') \qquad /3.9/$$

Summarizing these results:

$$f(s,t) = \eta\eta'\langle\tilde p_1,s_3,\lambda_3;\ \tilde p_2,s_4,-\lambda_4|\tau\ \cdot$$

$$\cdot\ \exp\left[-i\vartheta\left(\frac{\mu\sqrt{M^2-s}}{\sqrt{s(M^2-\mu^2)}}M_2^+ + \frac{M\sqrt{\mu^2-s}}{\sqrt{s(M^2-\mu^2)}}N_1^+\right)\right]|\tilde p_1,s_1,\lambda_1;\ \tilde p_2,s_2,-\lambda_2\rangle \qquad /3.10/$$

$$= \eta\eta'\langle\tilde p_1,s_3,\lambda_3;\ \tilde p_2,s_4,-\lambda_4|\tau G(s,\vartheta)|\tilde p_1,s_1,\lambda_1;\ \tilde p_2,s_2,-\lambda_2\rangle$$

where the $\exp(i\tilde\alpha N_3^+)$ factors of eq./3.5/ were put in the exponent of $\exp(-i\vartheta M_2)$.

As it is discussed elsewhere[5] $\vartheta/\sqrt{s}$ behaves regularly at s=o, hence eq./3.10/ can be continued to the non-positive s-region. It can be checked e.g. in the 4x4 representation that $G(\vartheta,s)$ of eq./3.10/ leaves invariant the total four-momentum of the two-particle states, i.e. it is a little-group element.

In the t and u channel ϑ differs from Re . To forbid complex parameters,what would mean some kind of complexification of the little-group, we separate it writing

$$G(s,\ \mathrm{Re}\vartheta + i\mathrm{Jm}\vartheta) = G(s,\ \mathrm{Re}\vartheta)\ G(s,\ i\ \mathrm{Jm}\vartheta)$$

and we cast $G(s,\ \mathrm{Re}\vartheta)$ onto the two-particle state in the right /or in the left/. Since

$$\left(M\sqrt{\mu^2-s}/\sqrt{|s|}\right)^2 - \left(\mu\sqrt{M^2-s}/\sqrt{|s|}\right)^2 = M^2-\mu^2 \qquad /3.11a/$$

we may introduce

$$\mathrm{ch}\gamma = M\sqrt{\mu^2-s}/\sqrt{|s|\left(M^2-\mu^2\right)},\quad \mathrm{sh}\gamma = \mu\sqrt{M^2-s}/\sqrt{|s|\left(M^2-\mu^2\right)} \qquad /3.11b/$$

Then

$$G(s,\ \mathrm{Re}\vartheta) = \exp\left[-i\pi\left(i\,\mathrm{ch}\,\gamma\ M_2^+ + i\ \mathrm{sh}\,\gamma\ N_1^+\right)\right] =$$

$$= \exp\left(i\gamma N_3^+\right)\ \exp\left(-i\left(i\pi N_1^+\right)\right)\ \exp\left(-i\gamma N_3^+\right) \qquad /3.12/$$

The effect of $\exp\left(-i\left(+i\,\pi N_1^+\right)\right)$ can be analyzed similarly, as we did for $\exp\left(-i\left(-i\,\pi N_3''\right)\right)$. This now changes the sign of the energy of both particles, leaving the helicities unchanged. The boosts in eq./3.12/ change the momenta of the individual particles, whereas the total four-momentum remains invariant.

4. CONCLUSION

In the foundation of the different theories the space of the particle states contains generally only physical states, i.e. states of positive energy. Accordingly, the scattering amplitude of a 1+2 → 3+4 process in the different channels is described by matrix elements

$f^s = \langle 3,4\ |1,2\rangle$, $f^t = \langle 2,4\ |1,3\rangle$, $f^u = \langle 2,3\ |1,4\rangle$ where $p_{oi} > o$, $i=1,2,3,4$.

These functions, f^s, f^t, f^u are connected with each other by analytic continuation.

In the considerations above we introduced - at least formally - states of negative energy; and we showed that the process of analytic continuation makes them plausible. In this approach the t and u channel amplitudes read as $f^u = \langle 3,-4 \mid 1,-2 \rangle$, $f^t = \langle -3,+4 \mid +1,-2 \rangle$, i.e. instead of writing e.g. $|1\rangle$ we use $\langle -1|$. /The minus sign refers only to the sign of the energy./ In other words the $\langle -1|$ "space" seems to be the dual one to the "space" $|1\rangle$.

The extension of this kind of the physical space shows some nice properties. We were able to write the amplitude as $\langle p_3, p_4 \mid TG \mid p_1, p_2 \rangle$, where G was always a little group element of p_1+p_2 and the group structure changed continuously along with $(p_1+p_2)^2$. Its merit is exploited elsewhere[5]. An other advantage is that the process shows some kind of "minimal complexification" of the Poincaré group, but we do not want to go into its details.

However, the extension of the space raises some questions what we can not answer yet; e.g. how the complete system looks like, how the extension of some operators should be defined, etc. We think, a possible approach to get answers is to examine the representations of the complexified Poincaré group.

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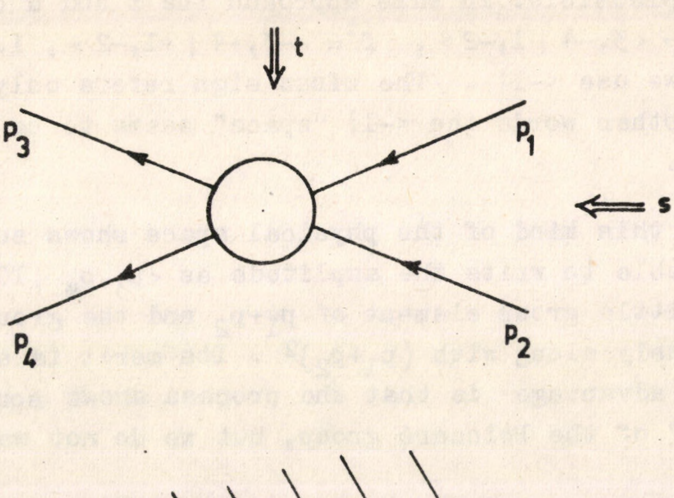
t
p3
p1
s
p4
p2

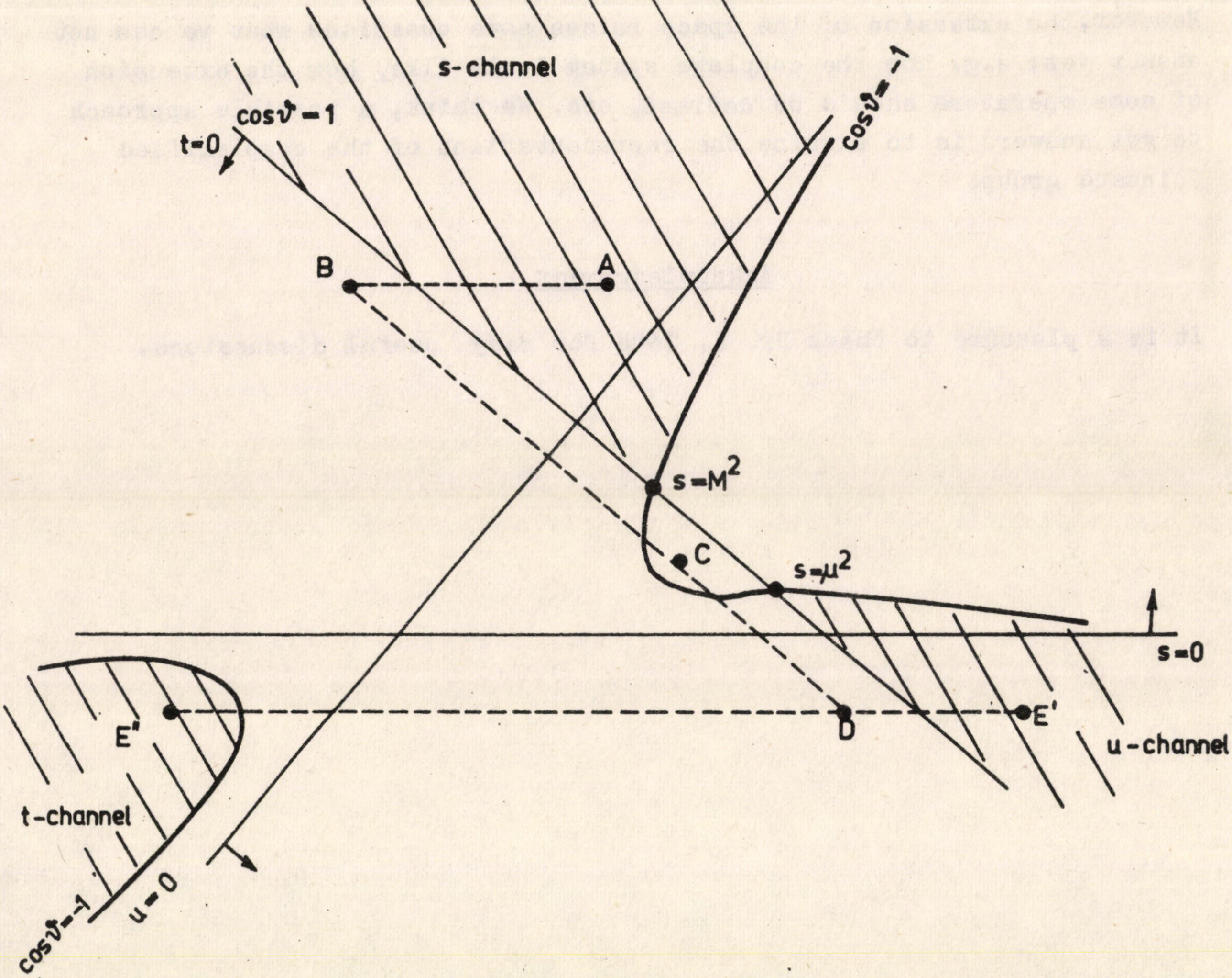
s-channel
cos ϑ=1
cos ϑ=-1
t=0
B
A
s=M²
C
s=μ²
s=0
E''
D
E'
u-channel
t-channel
u=0
cos ϑ=-1

Fig. 1

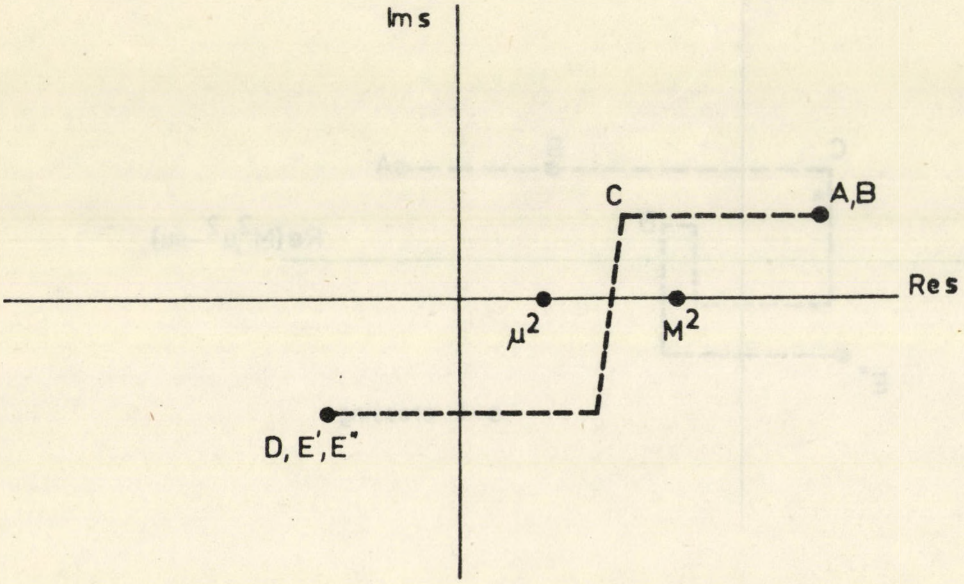
Im s
C
A,B
Re s
μ^2
M^2
D, E', E"

Fig. 2a

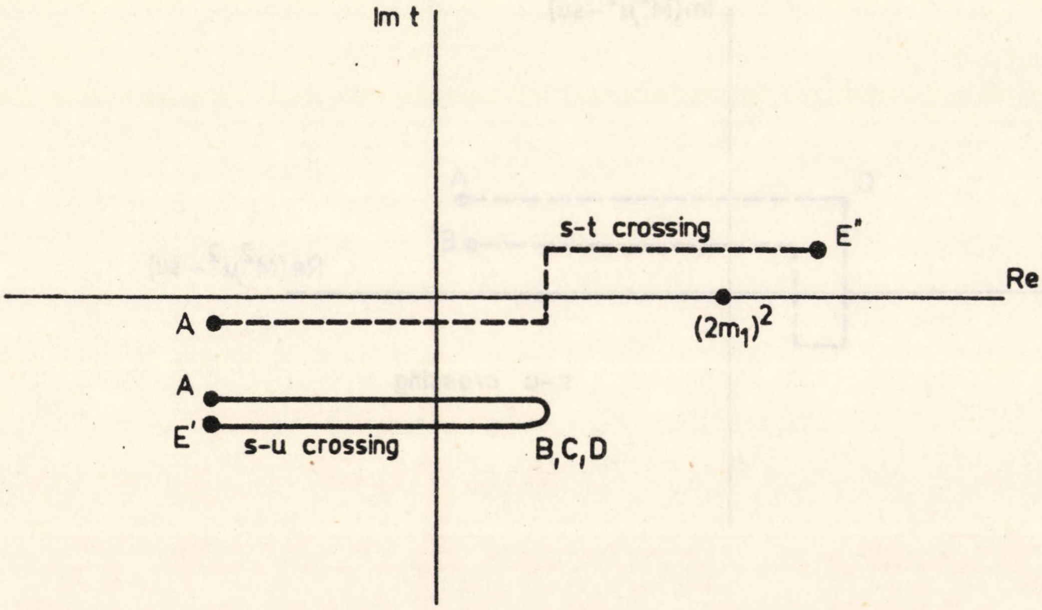
Im t
s-t crossing
E"
Re t
A
$(2m_1)^2$
A
E'
s-u crossing
B,C,D

Fig. 2b

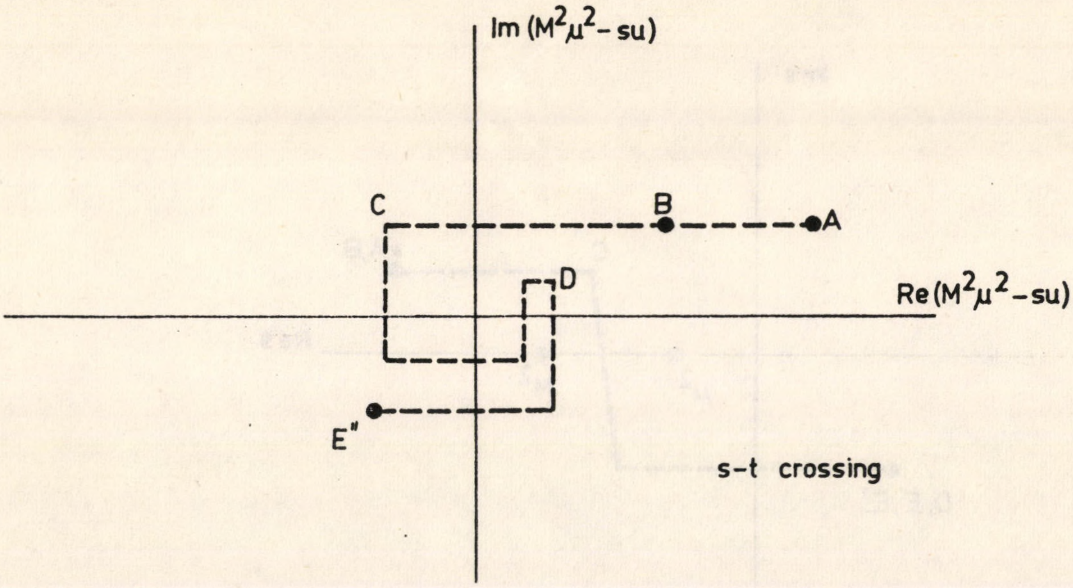
Im $(M^2\mu^2 - su)$
C
B
A
D
Re $(M^2\mu^2 - su)$
E''
s-t crossing

Fig. 3a

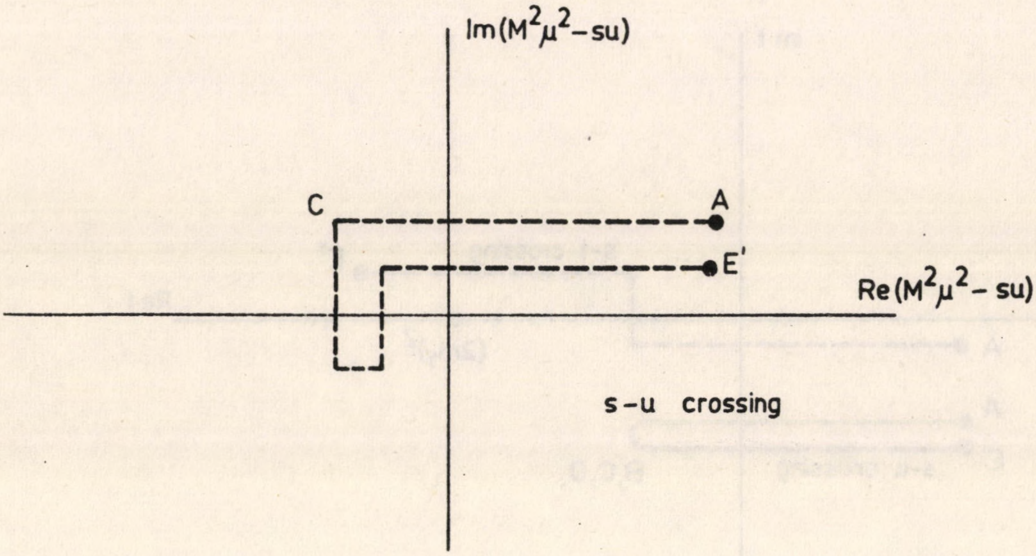
Im $(M^2\mu^2 - su)$
C
A
E'
Re $(M^2\mu^2 - su)$
s-u crossing

Fig. 3b

REFERENCES

[1] T.L. Trueman, G.Wick, Ann.of Phys. 26. 322. /1964/.

[2] L.L. Wang, Phys. Rev. 142. 1187. /1966/.

[3] M. Toller, Nuovo Cim. 54A 295. /1968/. J. Boyce et al,. preprint, IC/67/9.

[4] K. Szegő, K. Tóth, Nuovo Cim. 66A. 371. /1970/.

[5] K. Szegő, K. Tóth, to be published.

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